

Semester 1 (Unit 3) Examination, 2017

Question/Answer Booklet

MATHEMATICS SPECIALIST

Section One: Calculator-free

Student Name/Number: _____

Teacher Name: _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet

To be provided by the candidate:

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	10	10	100	92	65
					100

Instructions to candidates

- The rules for the conduct of School exams are detailed in the _____ *School/College assessment policy*.
Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer Booklet.
- You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
- Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.
- The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free**(52 Marks) Weighting 35%**

This section has 7 questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet.

Suggested working time: **50 minutes**.

Question 1**(9 marks)**

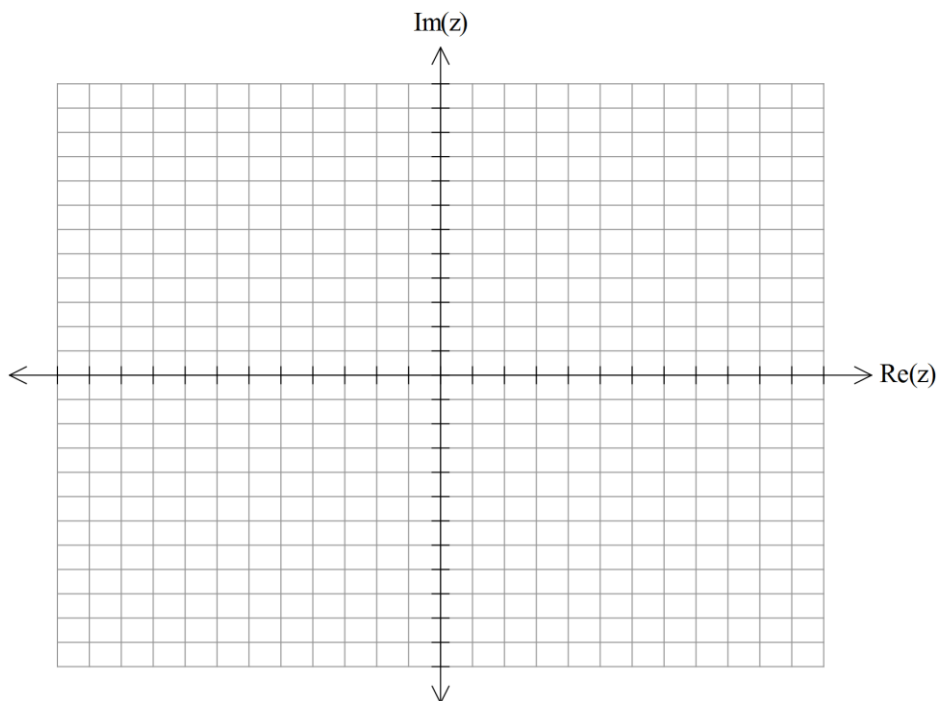
(a) If $z_1 = i$, $z_2 = 2 - 3i$ and $z_3 = k - i$, $k \in \mathbb{R}$, express each of the following in the form $a + bi$

(i) $z_1 - z_2$ (1 mark)

(ii) $z_3 \overline{z_2}$ (2 marks)

(iii) $\frac{z_1 z_2}{z_3}$ (3 marks)

- (b) On the diagram below, plot $q = 2 - 3i$, $w = 2i - 5$ and $k = 2 + (w - q)i$. (3 marks)

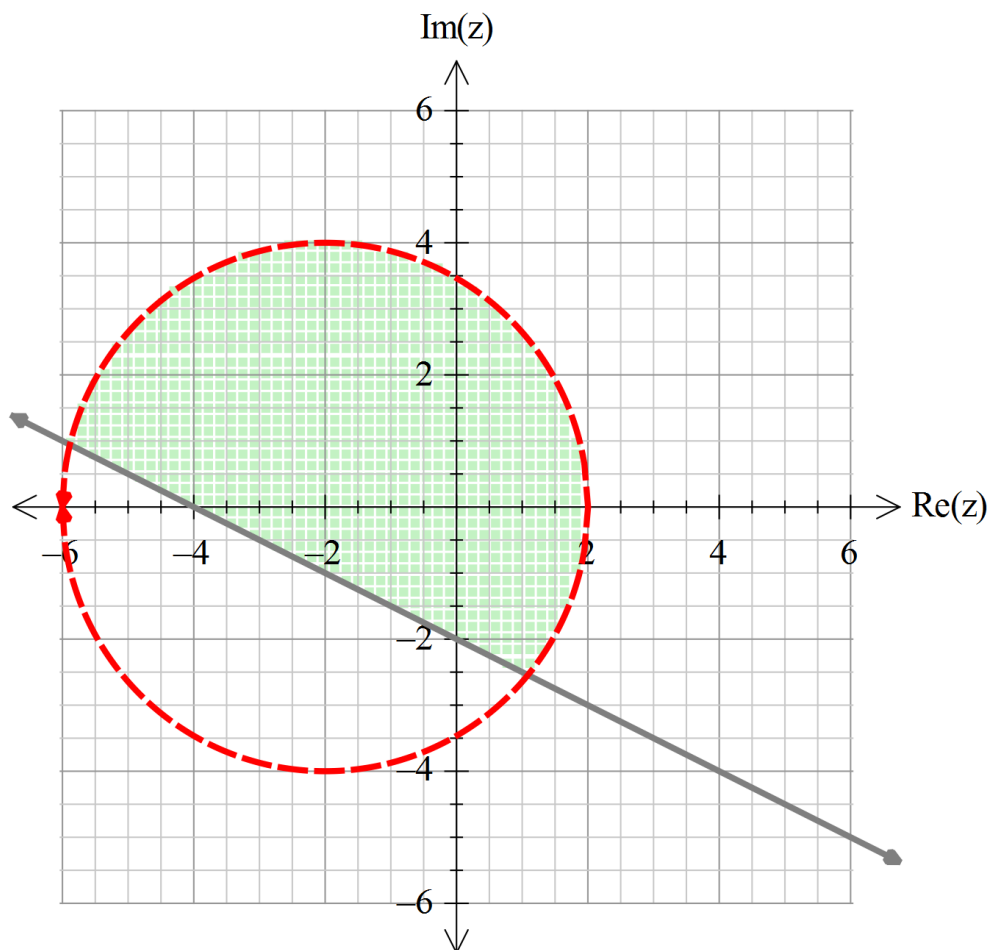


Question 2 (9 marks)

- (a) Express $z = 6cis\left(\frac{\pi}{3}\right)$ in Cartesian form. (2 marks)

- (b) Use de Moivre's theorem to determine z^4 , expressing your answer in Cartesian form. (3 marks)

- (c) State a symbolic description for the set of points indicated by the shaded region on the graph drawn below. (4 marks)



Question 3**(12 marks)**

Functions f and g are defined as follows $f(x) = \sqrt{x-3}$ and $g(x) = 2x+1$.

(a) Determine an expression for $f \circ g(x)$ and state its natural domain and range. (3 marks)

(b) Determine an expression for $g \circ f(x)$ and state its natural domain and range. (3 marks)

Function h is defined as $h(x) = 3x^2 - 12x + 11$.

(c) Determine the range of h over its natural domain. (2 marks)

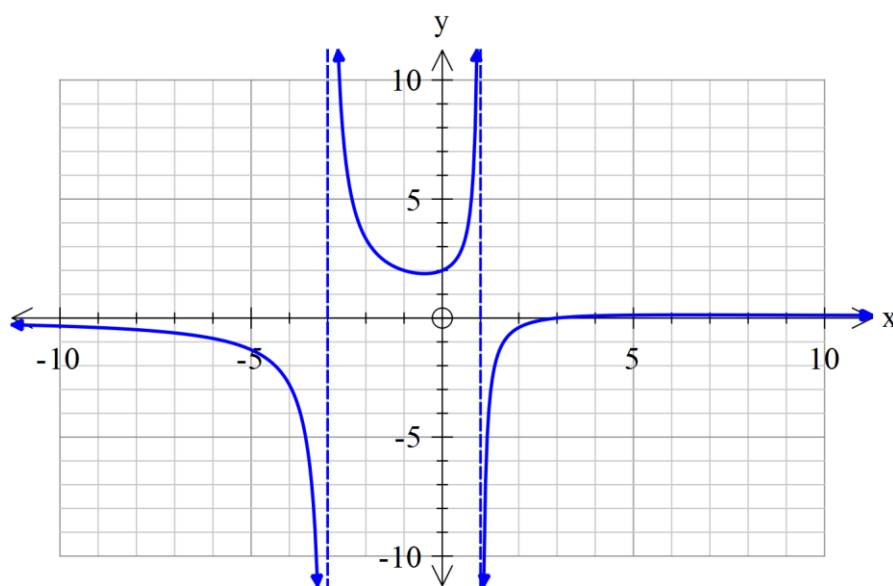
(d) The domain of h is restricted so that $y = h^{-1}(x)$ is a function and this restricted domain contains $x = 1$. If this restricted domain represents the largest possible domain, determine the defining rule for $y = h^{-1}(x)$. (4 marks)

Question 4

(3 marks)

The graph of the function $f(x) = \frac{ax+b}{(x+c)(x+d)}$ is shown on the axes below, where a, b, c and d are constants.

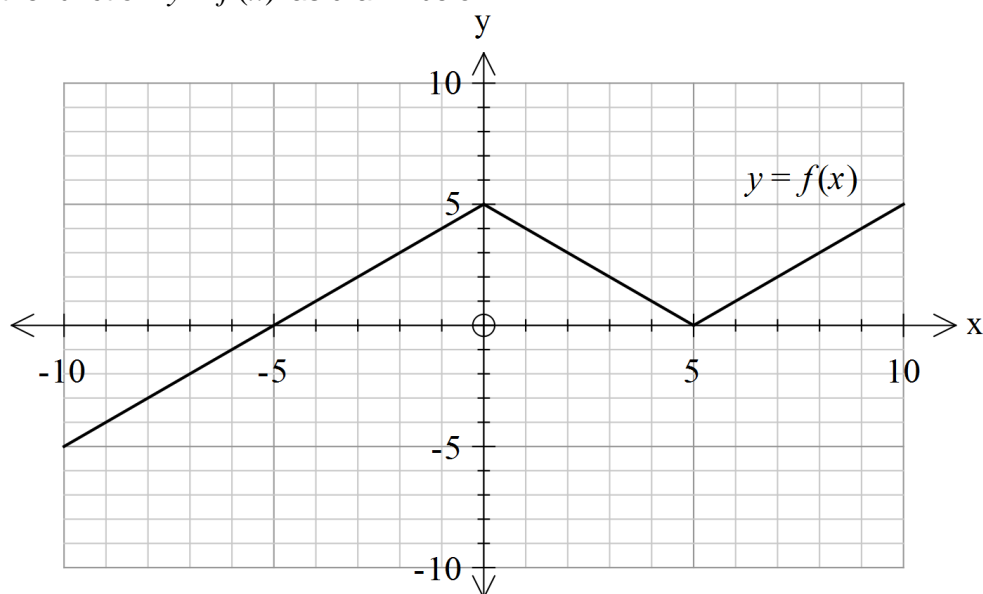
Determine the value of the constants given that $(0, 2)$ and $(-1, 2)$ are known points on this graph.



Question 5

(8 marks)

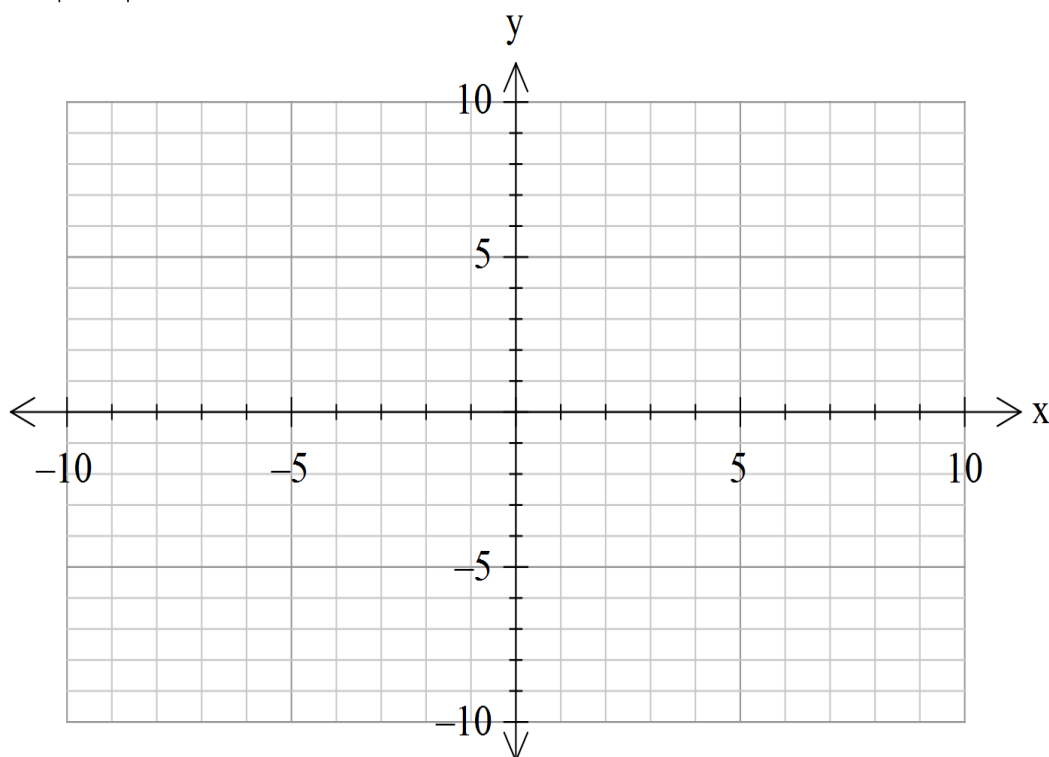
Consider the function $y = f(x)$ as drawn below.



On the axes below, sketch the following:

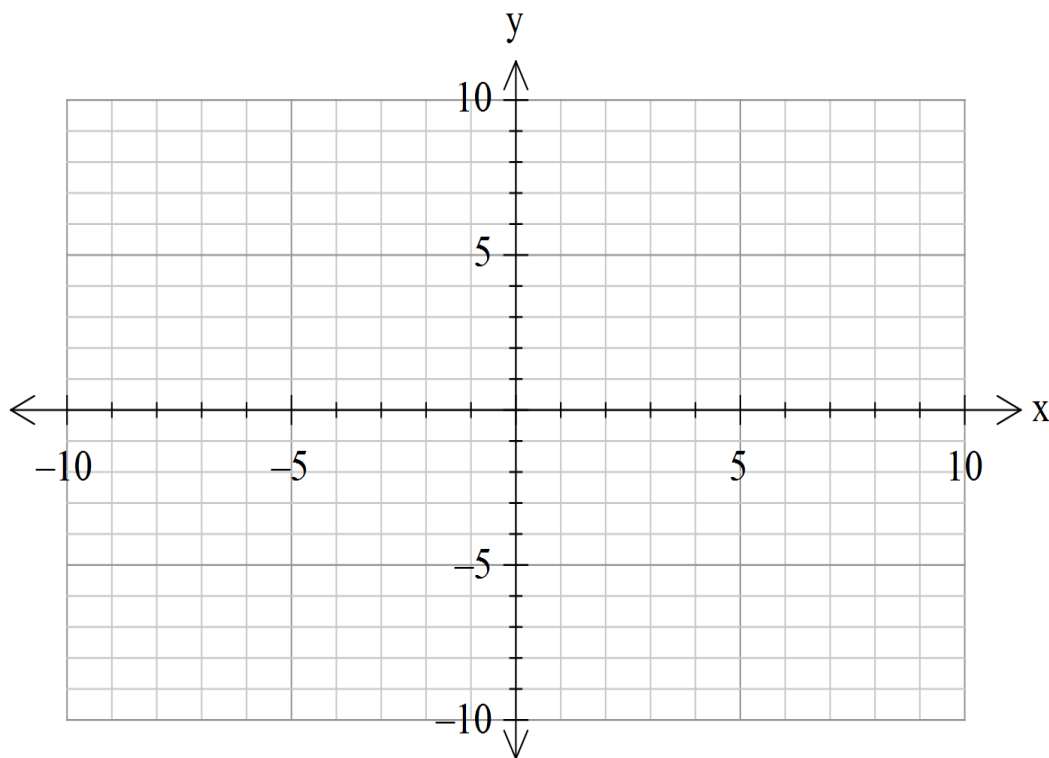
(a) $y = |f(x)|$

(2 marks)



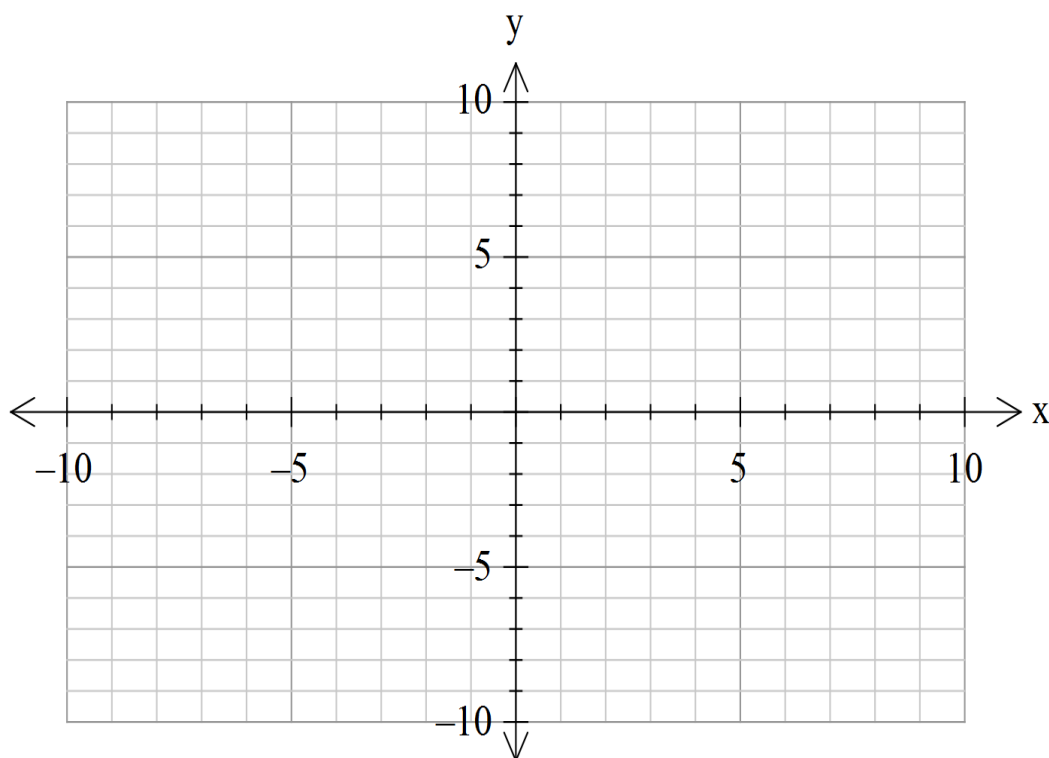
(b) $y = f(|x|)$

(2 marks)



(c) $y = \frac{1}{f(x)}$

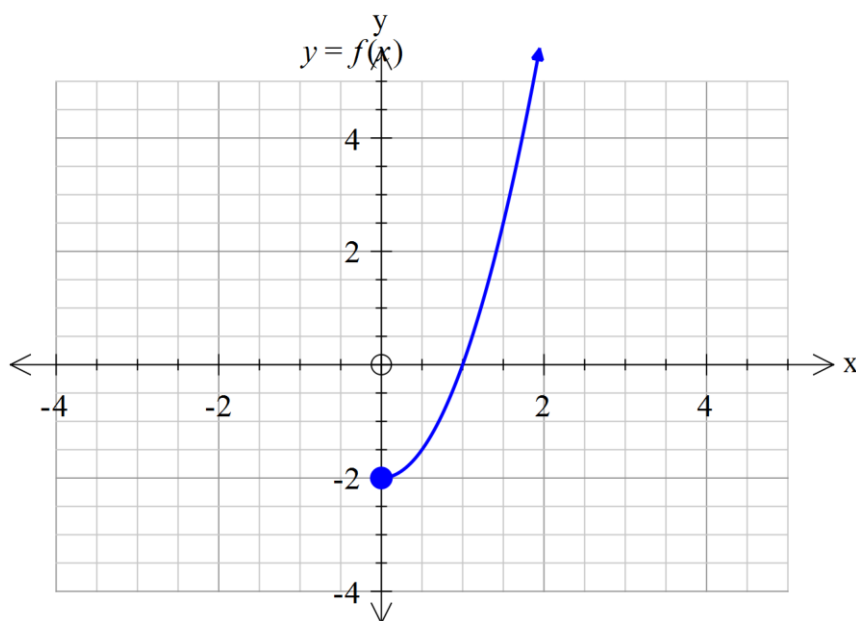
(4 marks)



Question 6

(5 marks)

Consider the function $f(x) = 2x^2 - 2$ with $x \geq 0$ which is drawn on the axes below.



(a) On the axes above, sketch $y = f^{-1}(x)$. (3 marks)

(b) Solve $f(x) = f^{-1}(x)$. (2 marks)

Question 7**(6 marks)**

(a) Solve the following system of linear equations:

(3 marks)

$$\begin{aligned}x + 3y - 2z &= 3 \\4x + 14y - 3z &= 19 \\3x + 12y + 2z &= 21\end{aligned}$$

(b) Consider the following system of equations:

(3 marks)

$$\begin{aligned}x + 3y - 2z &= 3 \\4x + 14y - 3z &= 19 \\3x + 12y + az &= b\end{aligned}$$

For which values of the constants a and b are there infinitely many solutions?

Additional working space

Question number: _____

Acknowledgements

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